KINEMATICS OF MACHINERY

DEPARTMENT OF MECHANICAL ENGINEERING

UNIT – I (SYLLABUS)

Mechanisms

- Kinematic Links and Kinematic Pairs
- Classification of Link and Pairs
- Constrained Motion and Classification

Machines

- Mechanism and Machines
- Inversion of Mechanism
- Inversions of Quadric Cycle
- Inversion of Single Slider Crank Chains
- Inversion of Double Slider Crank Chains

UNIT – II (SYLLABUS)

Straight Line Motion Mechanisms

- Exact Straight Line Mechanism
- Approximate Straight Line Mechanism
- Pantograph

Steering Mechanisms

- Davi’s Steering Gear Mechanism
- Ackerman’s Steering Gear Mechanism
- Correct Steering Conditions

Hooke’s Joint

- Single Hooke Joint
- Double Hooke Joint
- Ratio of Shaft Velocities

UNIT – III (SYLLABUS)

Kinematics

- Motion of link in machine
- Velocity and acceleration diagrams
- Graphical method
- Relative velocity method four bar chain

Plane motion of body

- Instantaneous centre of rotation
- Three centers in line theorem
- Graphical determination of instantaneous center

UNIT – IV (SYLLABUS)

Cams

- Cams Terminology
- Uniform velocity Simple harmonic motion
- Uniform acceleration
- Maximum velocity during outward and return strokes
- Maximum acceleration during outward and return strokes

Analysis of motion of followers

- Roller follower circular cam with straight
- concave and convex flanks

UNIT – V (SYLLABUS)

Gears

- Toothed gears types
- Condition for constant velocity ratio
- Velocity of sliding phenomena
- Condition for minimum number of teeth
- Expressions for arc of contact and path of contact

Gear Trains

- Simple and reverted wheel train
- Epicycle gear Train
- Differential gear for an automobile

COURSE OBJECTIVES

UNIT - 1	**To impart knowledge on various types of Mechanisms and synthesis**
UNIT - 2	To Synthesize and analyze 4 bar mechanisms
UNIT - 3	To impart skills to analyze the position, velocity and acceleration of mechanisms and synthesis of mechanism by analytical and graphical method
UNIT - 4	To familiarize higher pairs like cams and principles of cams design
UNIT - 5	To study the relative motion analysis and design of gears, gear trains

UNIT 1

CO1: To impart knowledge on various types of Mechanisms and synthesis

UNIT – I (SYLLABUS)

Mechanisms

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Machines

- Mechanism and Machines
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- Inversion of Single Slider Crank Chains
- Inversion of Double Slider Crank Chains

COURSE OUTLINE

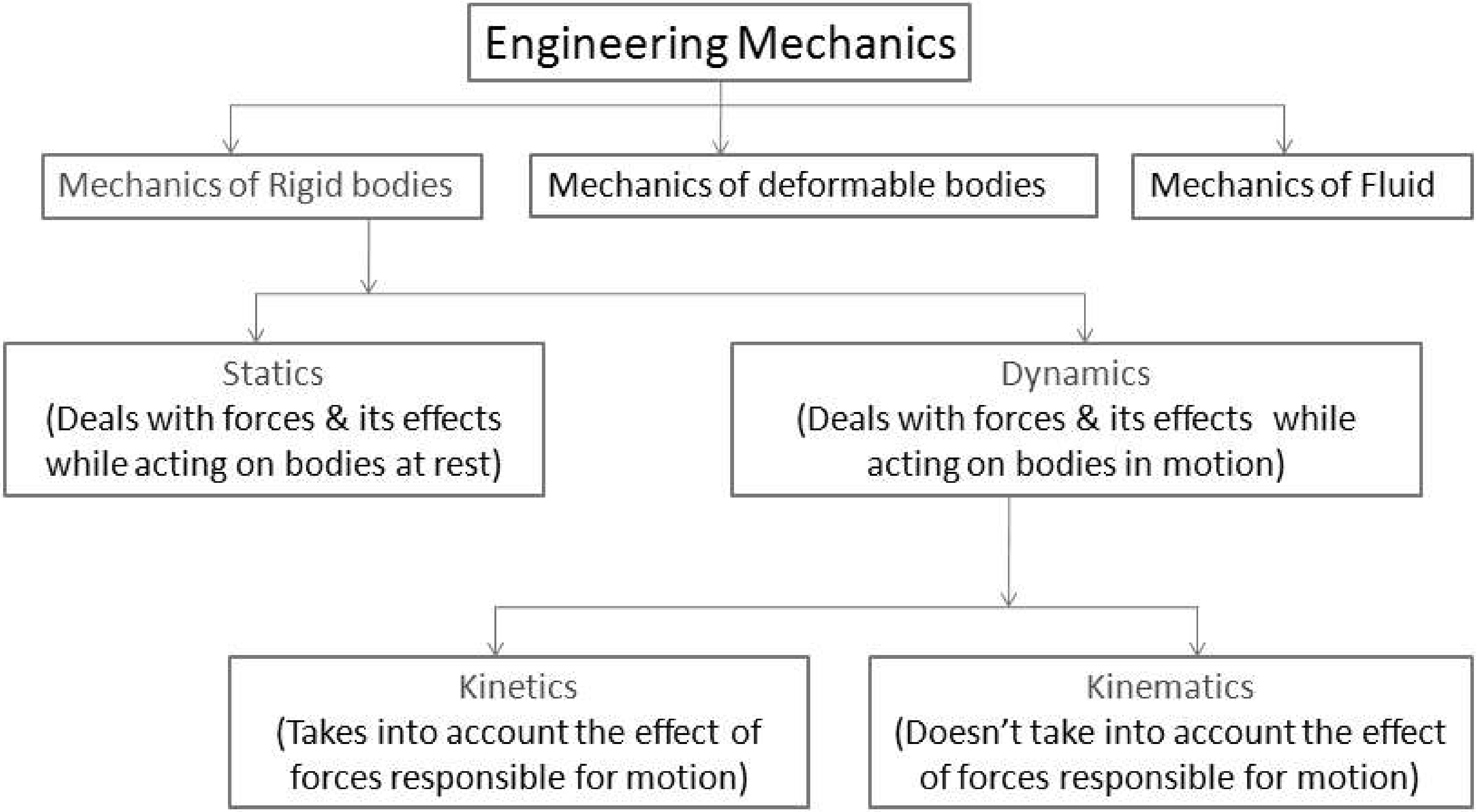
LECTURE	LECTURE TOPIC	KEY ELEMENTS	LEARNING OBJECTIVES
1	**Mechanisms**	Definition of Mechanism Definition of Machine	• Understanding the mechanics of rigid, fixed, deformable bodies (B2)
2	Kinematic Link and Classification of Links	Definition of Link Definition of Pair Classification of Links Classification of Pairs	• State the basic concept of link and pair (B1) • Understanding the classification of links and pairs (B2)
3	Constrained Motion and Classification	Definition of Constrained Motion Classification of Constrained Motion	• Describe the constrained motion (B1) • Understanding the direction of motion (B2)
4	**Mechanism and Machines**	Definition of Machine. Determine the nature of chain. Definition of Grashof’s law.	• Analyse machine and structure (B4) • Evaluate the nature of mechanism (B5)

COURSE OUTLINE

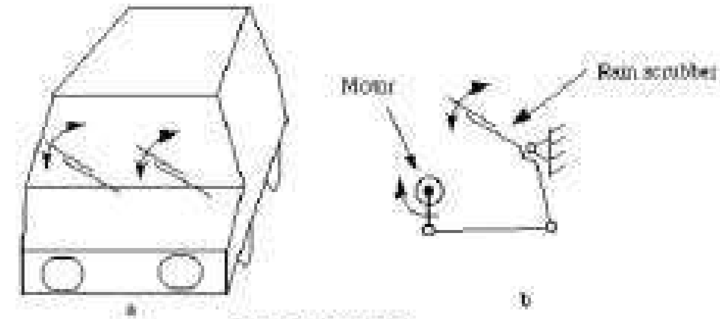
LECTURE	LECTURE TOPIC	KEY ELEMENTS	LEARNING OBJECTIVES
5	Inversion of Mechanism	• Definition of inversion. • Classification of inversion of mechanism	• Understanding the inversion of mechanisms and its classifications (B2)
6	Inversions of Quadric Cycle	• Working of 4-bar chain mechanisms	• Understanding the important inversions of 4-bar mechanism (B2) • Analyse the inversion of 4-bar mechanism (B4)
7	Inversion of Single Slider Crank Chains	• Working of Single slider crank chain mechanisms	• Understanding the important inversions of single mechanism (B2) • Analyse the inversion of single mechanism (B4)
8	Inversion of Double Slider Crank Chains	• Working of Double slider crank chain mechanisms	• Understanding the important inversions of single mechanism (B2)

LECTURE 1

Mechanisms


Engineering Mechanics
Mechanics of Rigid bodies
Mechanics of deformable bodies
Mechanics of Fluid
Statics
(Deals with forces & its effects while acting on bodies at rest)
Dynamics
(Deals with forces & its effects while acting on bodies in motion)
Kinetics
(Takes into account the effect of forces responsible for motion)
Kinematics
(Doesn't take into account the effect of forces responsible for motion)

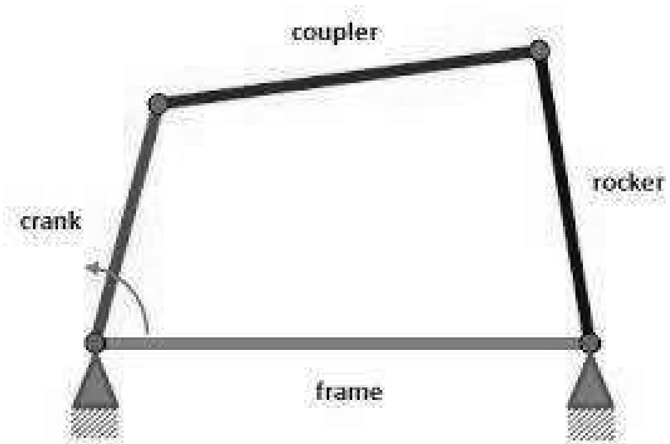
BASICS

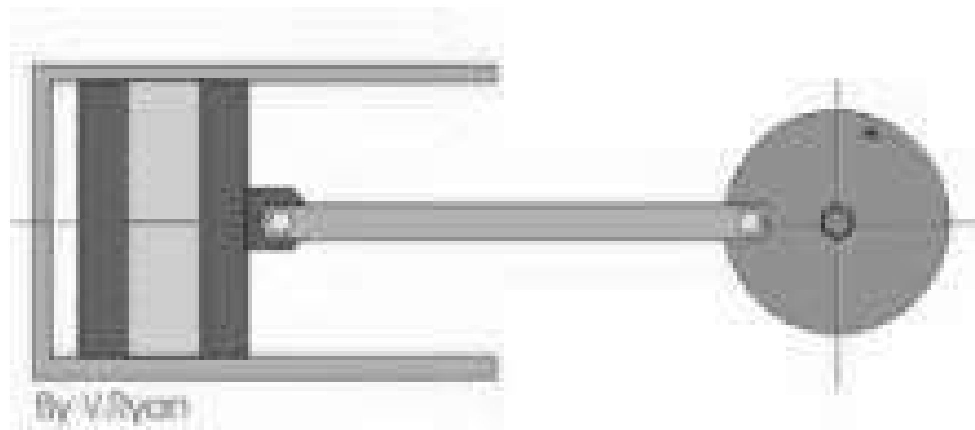


Windshield wiper

Mechanism:

- A number of bodies are assembled in such a way that the motion of one causes constrained and predictable motion to the others.
- A mechanism <u>transmits and modifies</u> a motion.
- Example: 4 bar mechanism, Slider crank mechanism

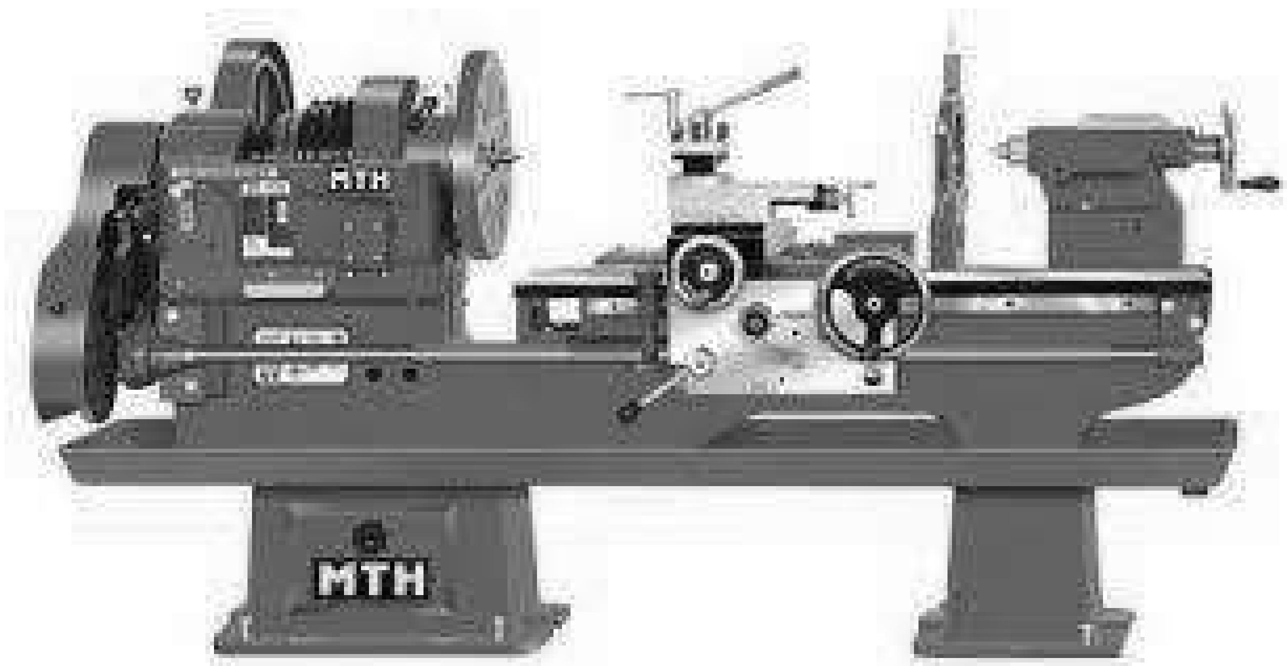




BASICS

Machine: (Combinations of Mechanisms)

Transforms energy available in one form to another to do certain type of desired useful work.

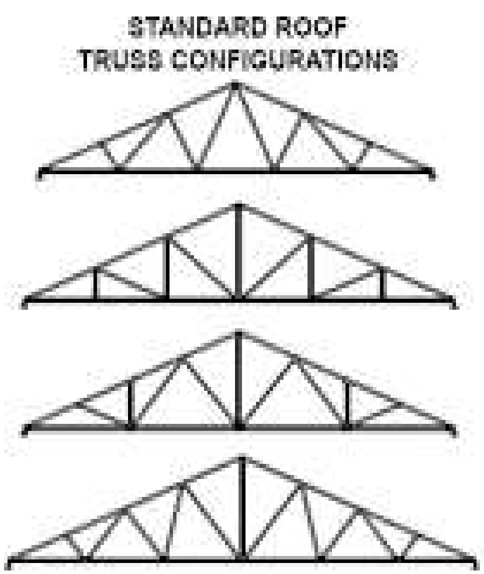


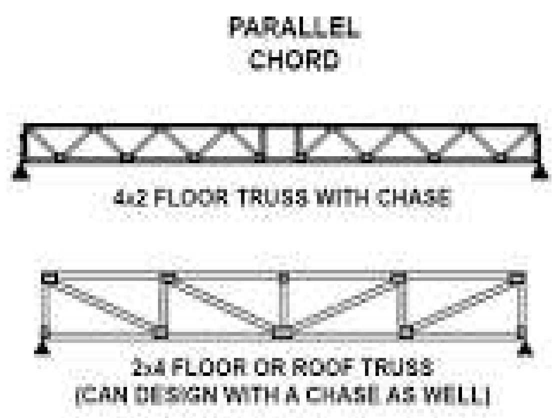
Lathe Machine

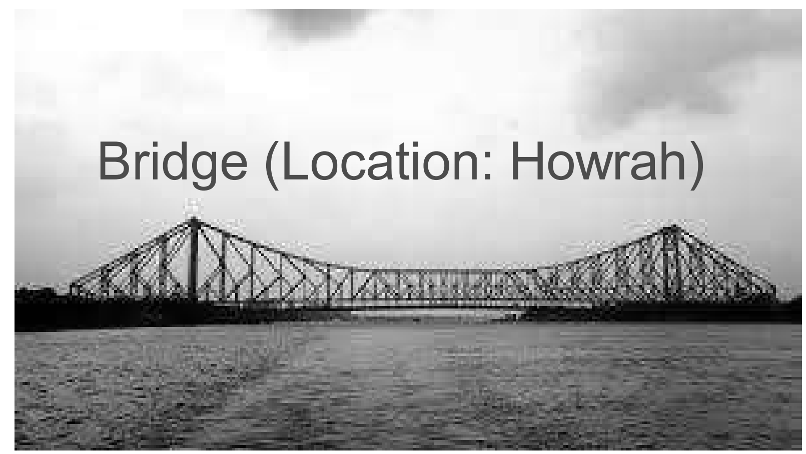
BASICS

Structure:

- ➢ Assembly of a number of resistant bodies meant to take up loads.
- ➢ No relative motion between the members






Bridge (Location: Howrah)

Truss

LECTURE 2

KINEMATIC LINK AND CLASSIFICATION OF LINKS

BASICS

Kinematic Link (element): It is a Resistant body i.e. transmitting the required forces with negligible deformation.

Types of Links

1. Rigid Link

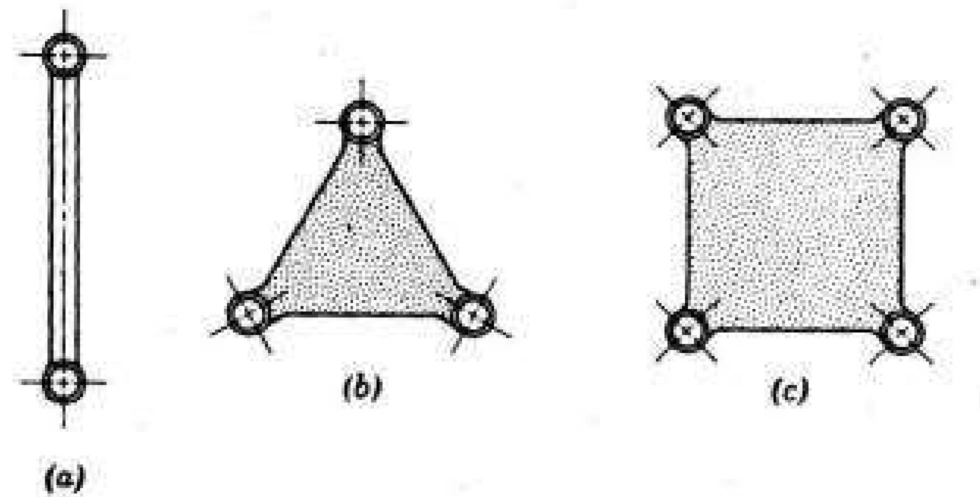
Doesn't undergo deformation. Example: Connecting rod, crank

2. Flexible Link

Partially deformed link. Example: belts, Ropes, chains

3. Fluid Link

Formed by having a fluid in a receptacle and the motion is transmitted through the fluid by pressure or compression only. Example: Jacks, Brakes



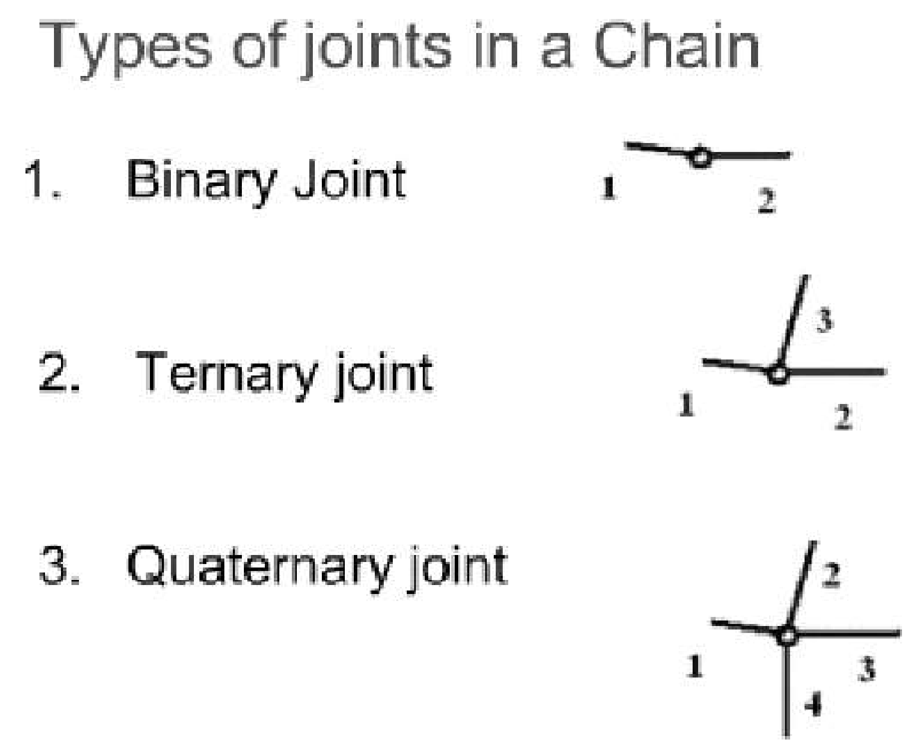
Binary link (2 vertices) **Ternary link (3 vertices)** **Quaternary link (4 vertices)**

BASICS

Kinematic Joint: Connection between two links by a pin

Types of Joints:

- Binary Joint (2 links are connected at the joint)
- Ternary Joint (3 links are connected)
- Quaternary Joint. (4 links are connected)

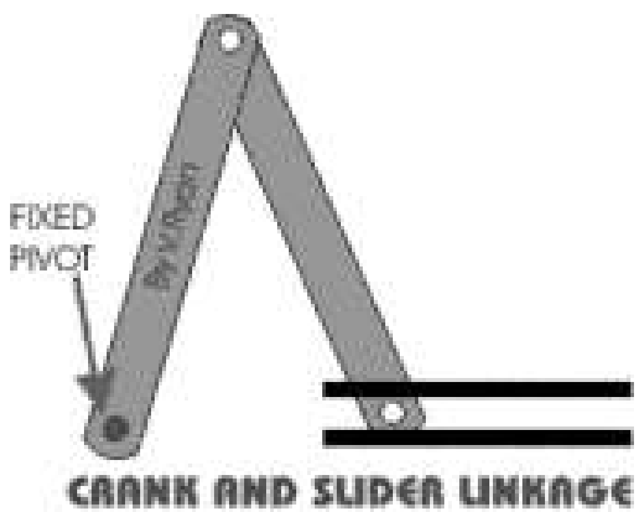


Note: if ‘l’ number of links are connected at a joint, it is equivalent to (l-1) binary joints.

BASICS

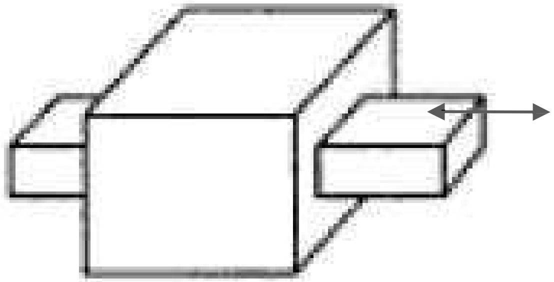
Kinematic Pair:

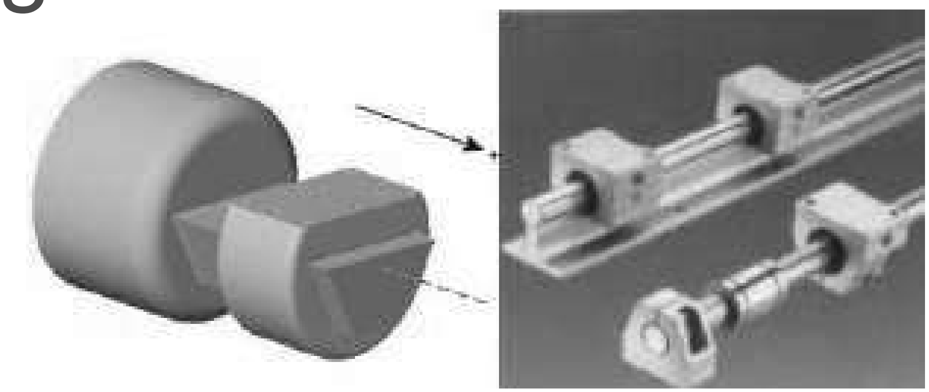
- The two links (or elements) of a machine, when in contact with each other, are said to form a pair.
- If the relative motion between them is completely or successfully constrained (i.e. in a definite direction), the pair is known as kinematic pair



KINEMATIC PAIRS ACCORDING TO THE RELATIVE MOTION

1. Sliding Pair





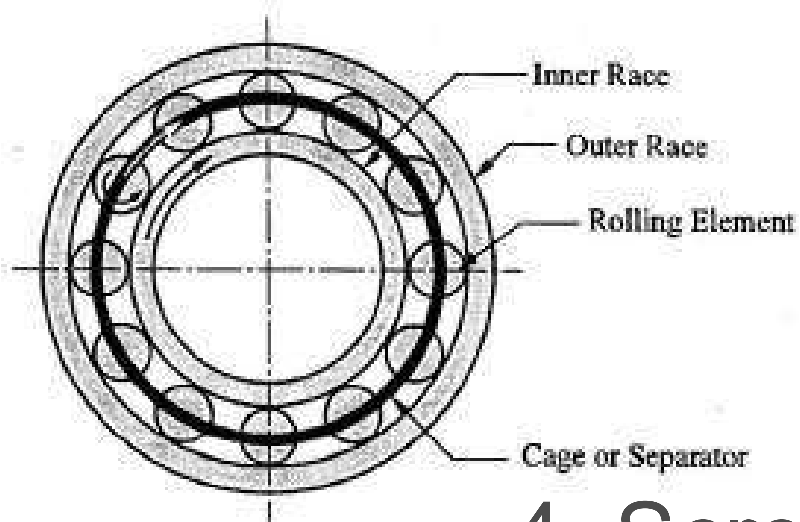
Rectangular bar in a rectangular hole

2. Turning or Revolving Pair

Collared shaft revolving in a circular hole

KINEMATIC PAIRS ACCORDING TO THE RELATIVE MOTION

3. Rolling Pair



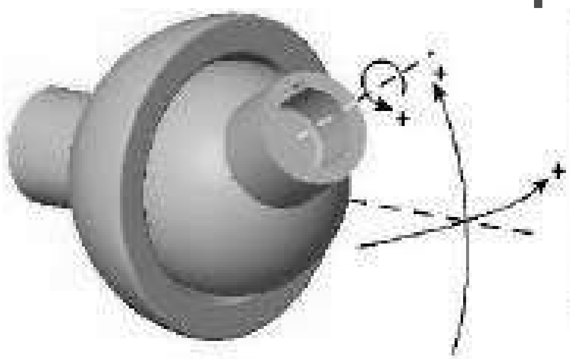
Links of pairs have a rolling motion relative to each other.

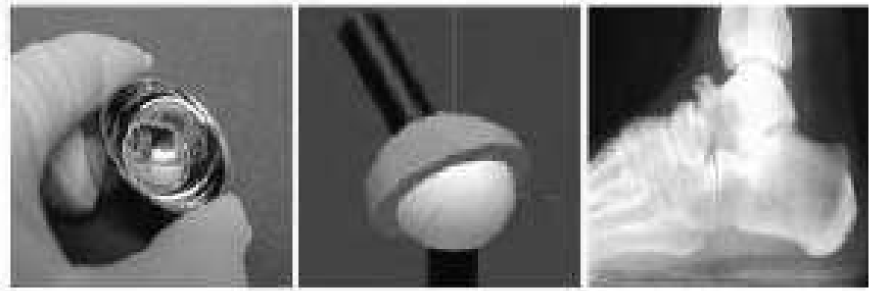
4. Screw or Helical Pair

if two mating links have a turning as well as sliding motion between them.

KINEMATIC PAIRS ACCORDING TO THE RELATIVE MOTION

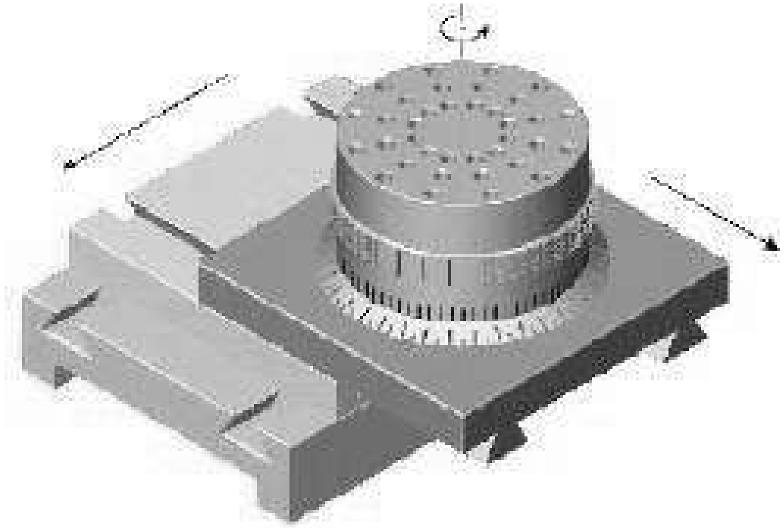
5. Spherical Pair

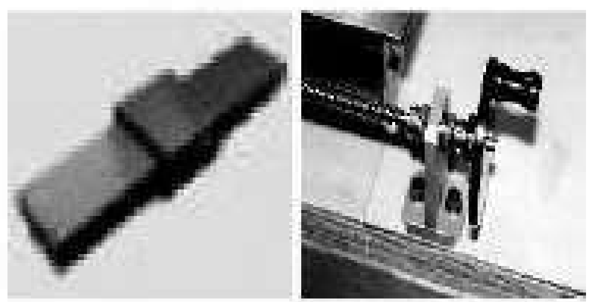




When one link in the form of a sphere turns inside a fixed link

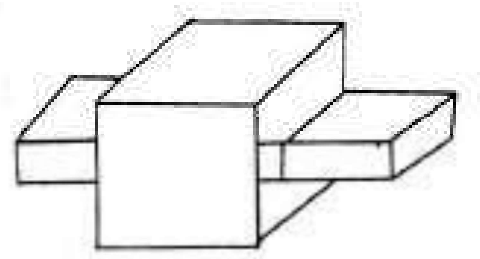
6. Planar Pair

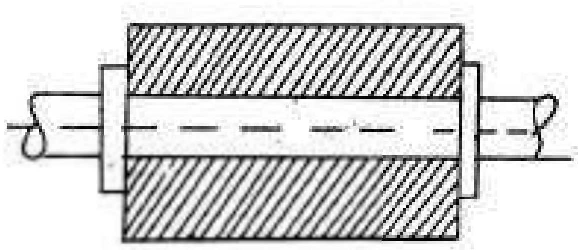




KINEMATIC PAIRS ACCORDING TO TYPE OF CONTACT

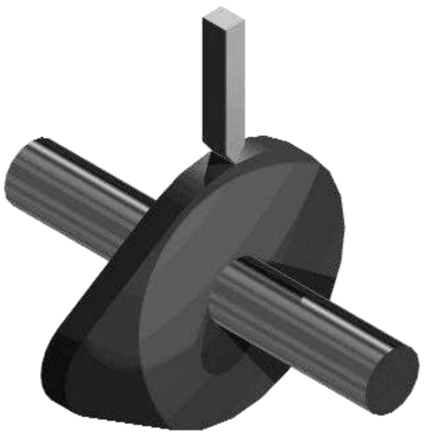
1. Lower Pair





The joint by which two members are connected has surface (Area) contact

2. Higher Pair



The contact between the pairing elements takes place at a point or along a line.

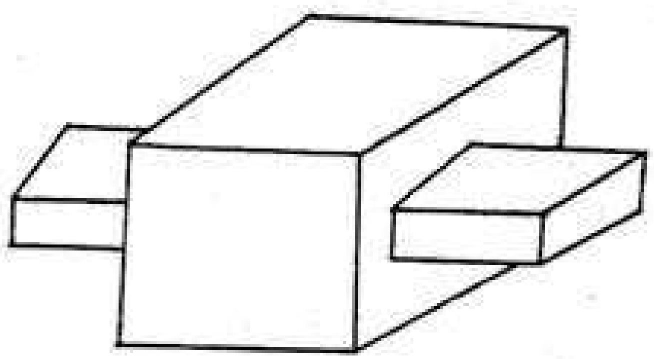
Toothed gearing, belt and rope drives, ball and roller bearings and cam and follower are the examples of higher pairs

LECTURE 3

CONSTRAINED MOTION AND CLASSIFICATION

KINEMATIC PAIRS ACCORDING TO TYPE OF CONSTRAINT

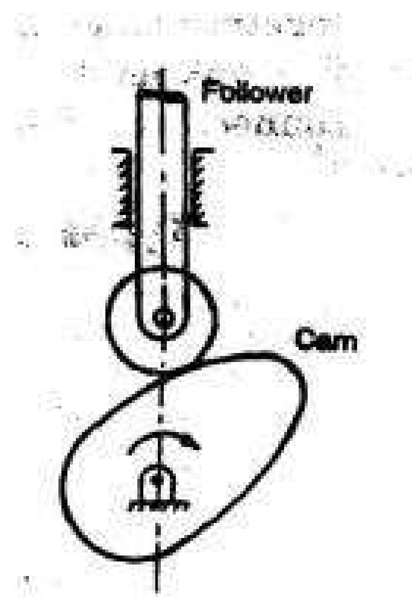
1. Closed Pair



Two elements of pair are held together mechanically to get required relative motion.

Eg. All lower pairs

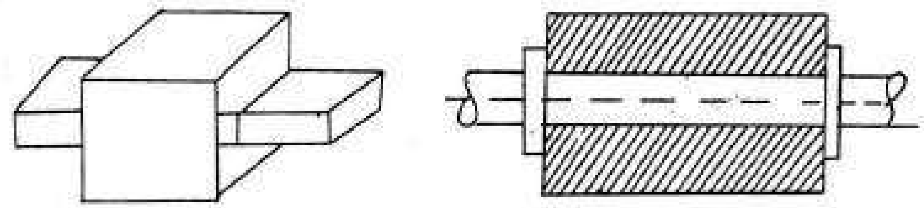
2. Unclosed Pair



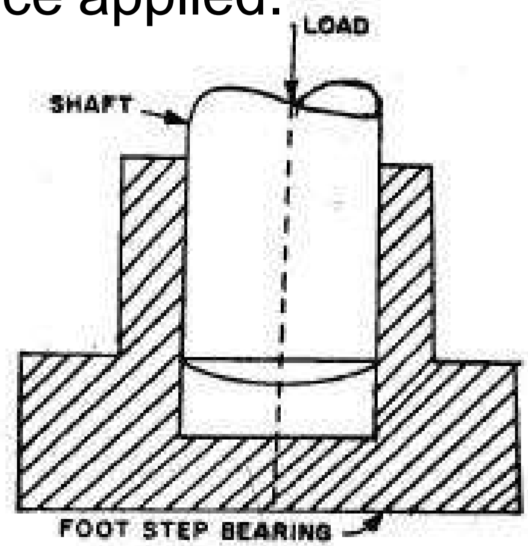
- Elements are not held mechanically.
- Held in contact by the action of external forces.

Eg. Cam and spring loaded follower pair

CONSTRAINED MOTION

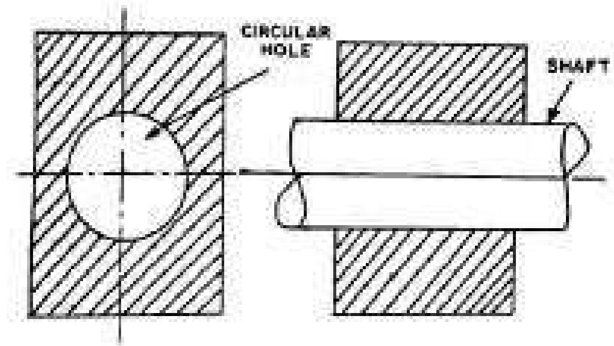


1. Completely constrained Motion:
Motion in definite direction irrespective of the direction of the force applied.



2. Successfully (partially) constrained Motion:

- Constrained motion is not completed by itself but by some other means.
- Constrained motion is successful when compressive load is applied on the shaft of the foot step bearing



3. Incompletely constrained motion:
Motion between a pair can take place in more than one direction.

Circular shaft in a circular hole may have rotary and reciprocating motion. Both are independent of each other.

KINEMATIC CHAIN

Group of links either joined together or arranged in a manner that permits them to move relative (i.e. completely or successfully constrained motion) to one another.

Example: 4 bar chain

The following relationship holds for kinematic chain

$$l = 2p - 4$$

$$j = \frac{3}{2}l - 2$$

Where

p = number of lower pairs

l = number of links

j = Number of binary joints

KINEMATIC CHAIN

$$l = 2p - 4$$

$$j = \frac{3}{2}l - 2$$

If LHS > RHS, Locked chain or redundant chain; no relative motion possible.

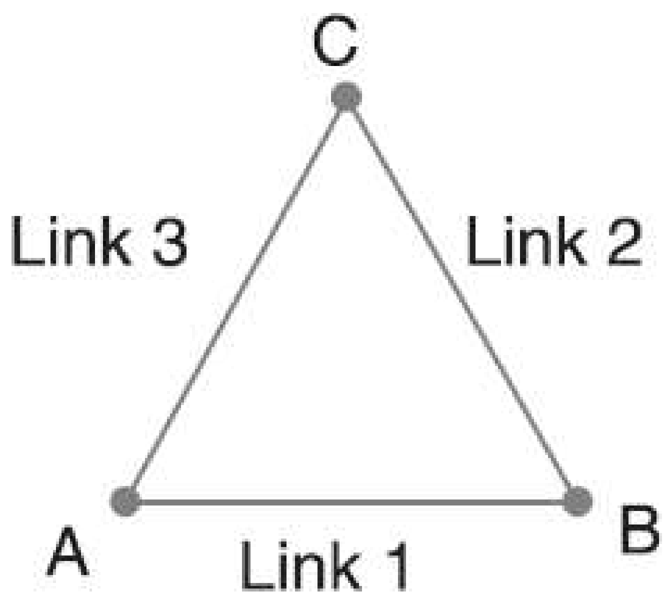
LHS = RHS, Constrained chain .i.e. motion is completely constrained

LHS < RHS, unconstrained chain. *i.e. the relative motion is not completely constrained.*

NUMERICAL EXAMPLE-1

Determine the nature of the chain

(K2:U)



$l = 3 \quad p = 3 \quad j = 3$

From equation

$$l = 2p - 4$$
$$= 2 \times 3 - 4 = 2$$

L.H.S. > R.H.S.

$$j = \frac{3}{2} l - 2$$
$$= \frac{3}{2} \times 3 - 2 = 2.5$$

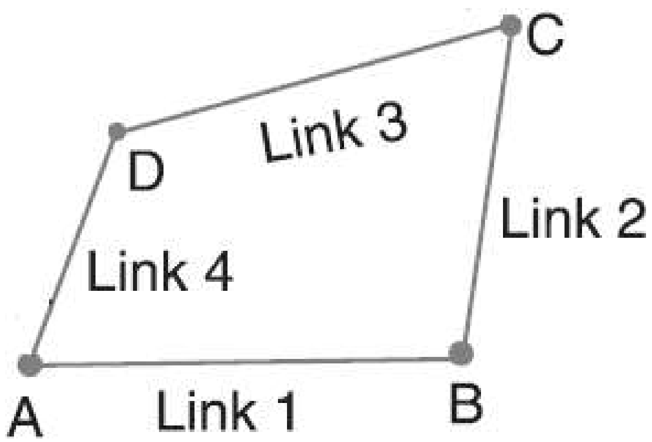
L.H.S. > R.H.S.

Therefore it is a locked Chain

EXERCISE

Determine the nature of the chains given below (K2:U)

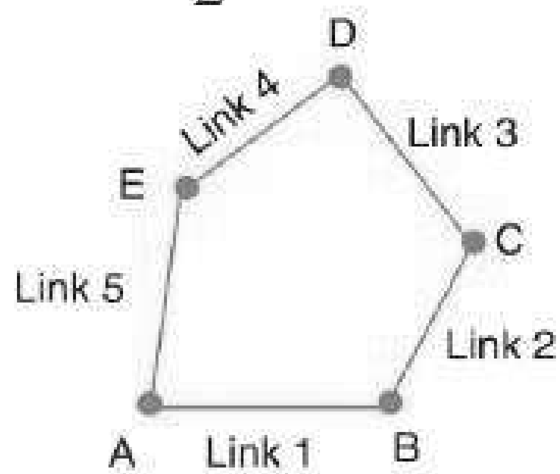
Hint: Check equations $l = 2p - 4$, $j = \frac{3}{2}l - 2$



$l = 4$, $p = 4$, and $j = 4$

L.H.S. = R.H.S.

constrained kinematic chain



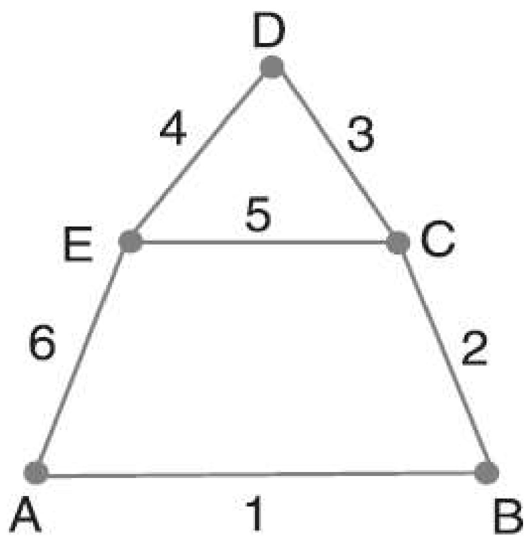
$l = 5$, $p = 5$, and $j = 5$

L.H.S. < R.H.S.

unconstrained chain

NUMERICAL EXAMPLE-2

Determine the nature of the chain (K2:U)



- **l** = 6
- j = 3 Binary joints (A, B & D) + 2 ternary joints (E & C)
- We know that, 1 ternary joint = (3-1) = 2 Binary Joints
- Therefore, j = 3 + (2*2) = 7.

$$j = \frac{3}{2} l - 2$$

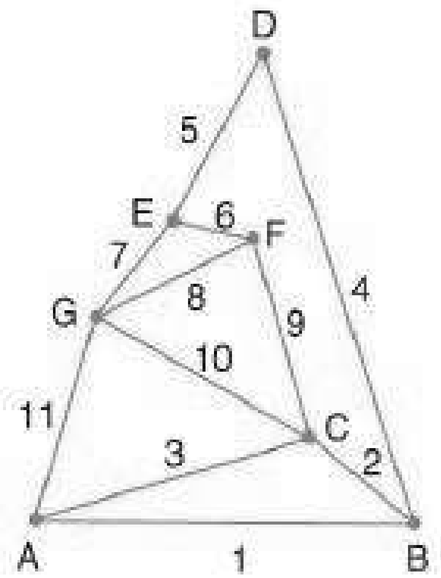
$$= \frac{3}{2} \times 6 - 2 = 7$$

L.H.S. = R.H.S.

Therefore, the given chain is a kinematic chain or constrained chain.

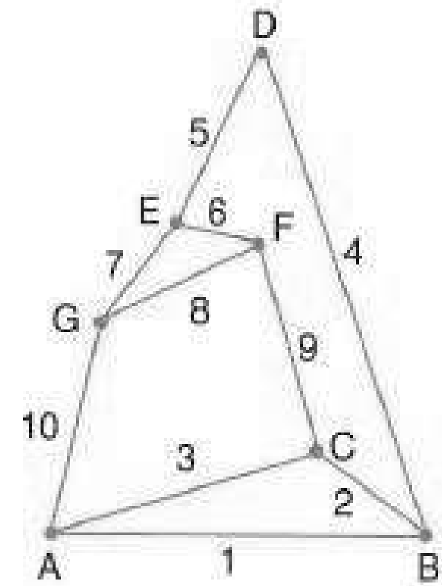
EXERCISE

Determine the number of joints (equivalent binary) in the given chains (K2:U)



Number of Binary Joints = 1 (D)
No. of ternary joints = 4 (A, B, E, F)
No. of quaternary joints = 2 (C & G)

Therefore, j = 1 + 4 (2) + 2 (3)
= 15



No. of Binary Joints = 1 (D)
No. of ternary joints = 6 (A, B, C, E,F, G)

j = 1 + 6 (2) = 13

KINEMATIC CHAIN

- For a kinematic chain having higher pairs, each higher pair is taken equivalent to two lower pairs and an additional link.
- In this case to determine the nature of chain, the relation given by A.W. Klein, may be used

$$j + \frac{h}{2} = \frac{3}{2} l - 2$$

where j = Number of binary joints,

h = Number of higher pairs, and

l = Number of links.

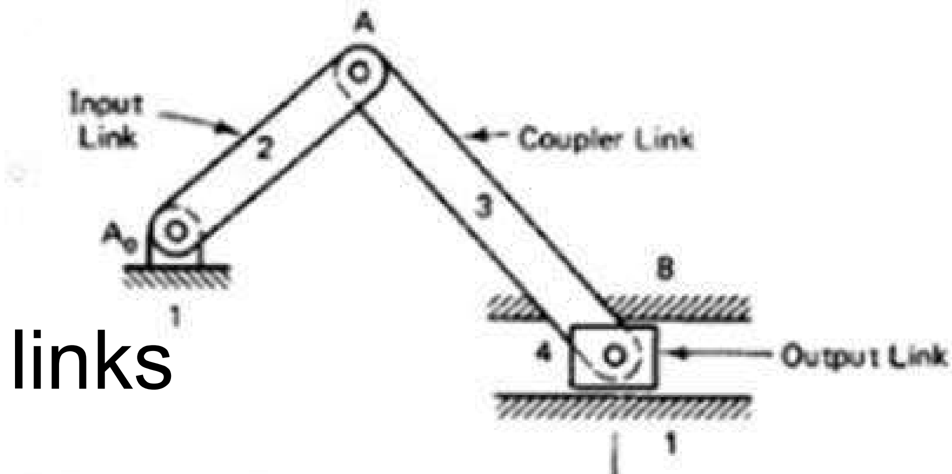
LECTURE 4

MECHANISM AND MACHINES

CLASSIFICATION OF MECHANISMS

Mechanism:

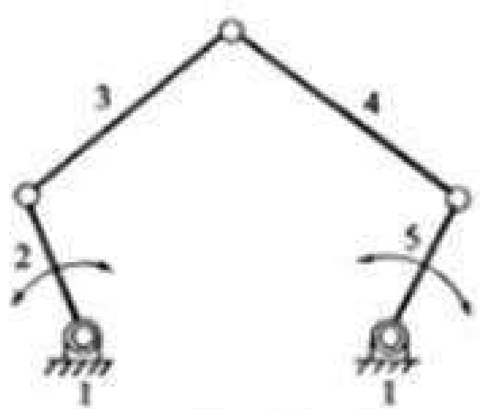
When one of the links of a kinematic chain is fixed, the chain is called Mechanism.



Simple Mechanism

Types:

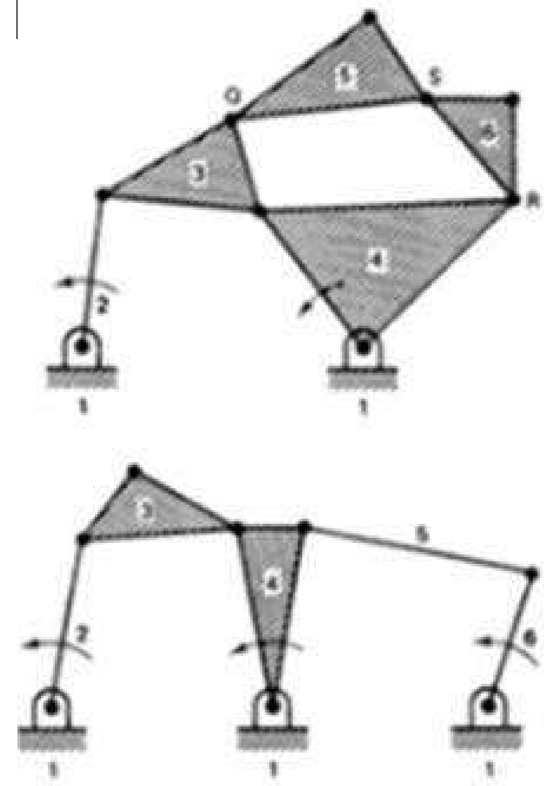
- Simple - 4 Links
- Compound - More than 4 links



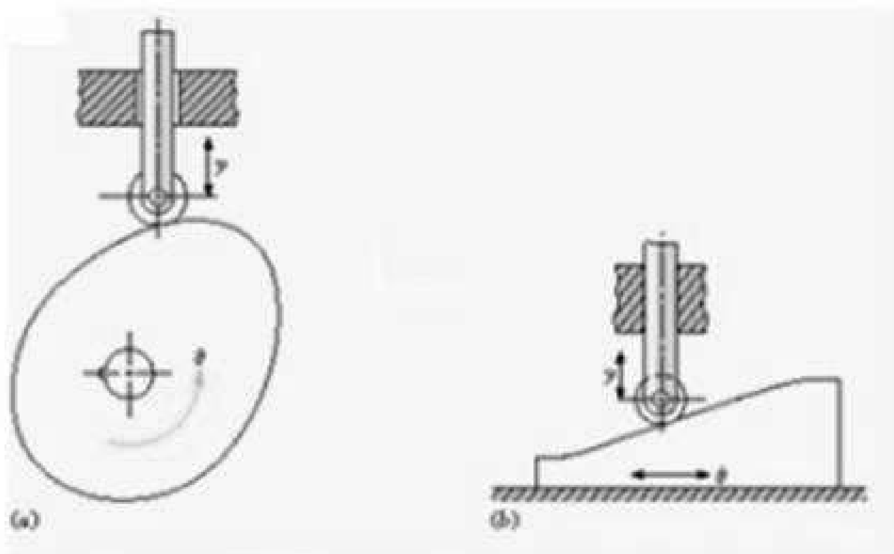
Compound Mechanism

Classification of mechanisms

- Complex - Ternary or Higher order floating Links
- Planar - All links lie in the same plane



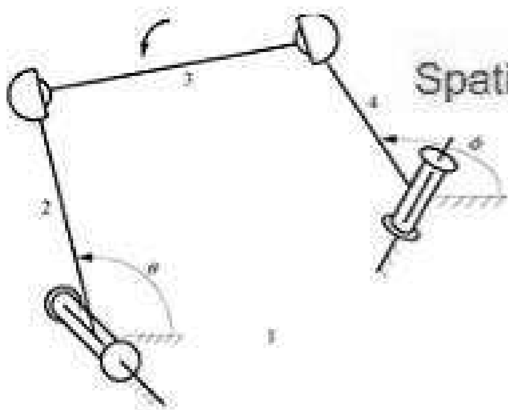
Complex Mechanism



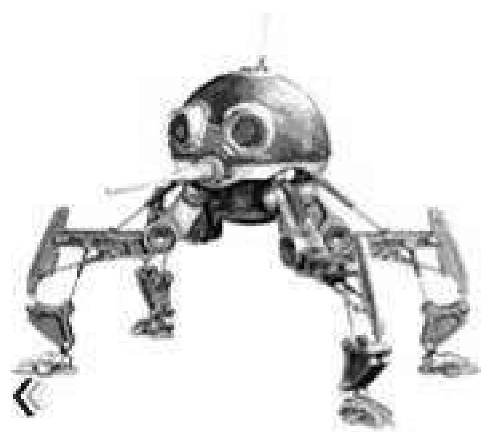
Planar Mechanism

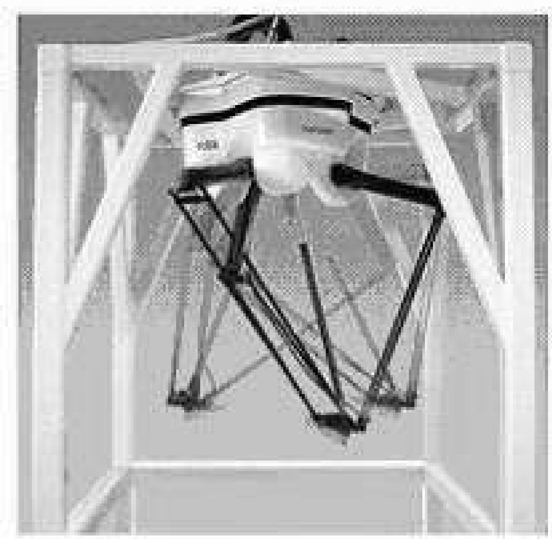
Classification of mechanisms

➢Spatial - Links of a mechanism lie in different planes



Spatial Mechanism





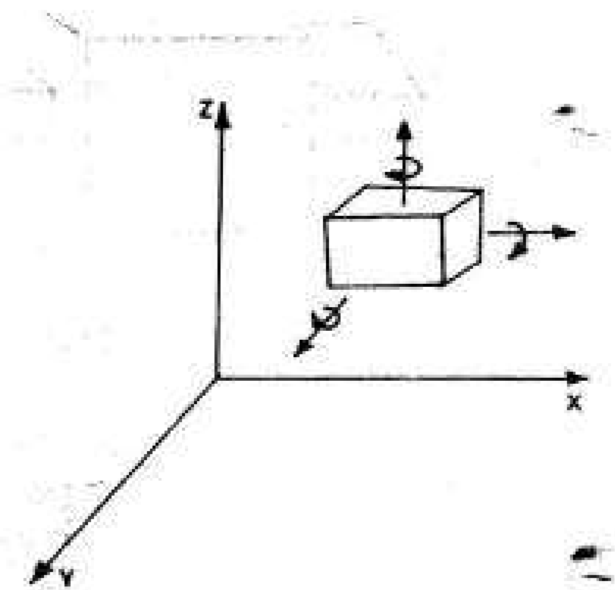
Parallel robot

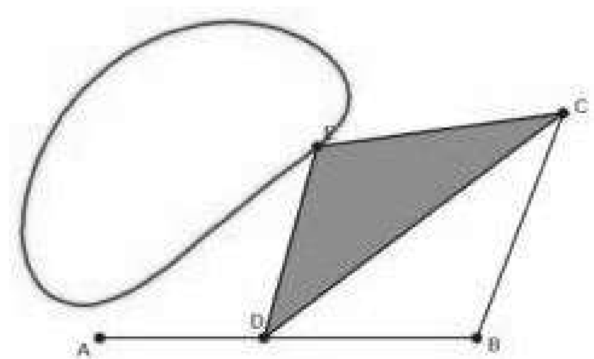
Machine

When a <u>mechanism</u> is required to <u>transmit power</u> or to <u>do some particular type of work</u>, it then becomes a **machine**. In such cases, the various links or elements have to be designed to withstand the forces (both static and kinetic) safely.

DEGREES OF FREEDOM (DOF) / MOBILITY

It is the number of **independent coordinates** required to describe the **position of a body**.

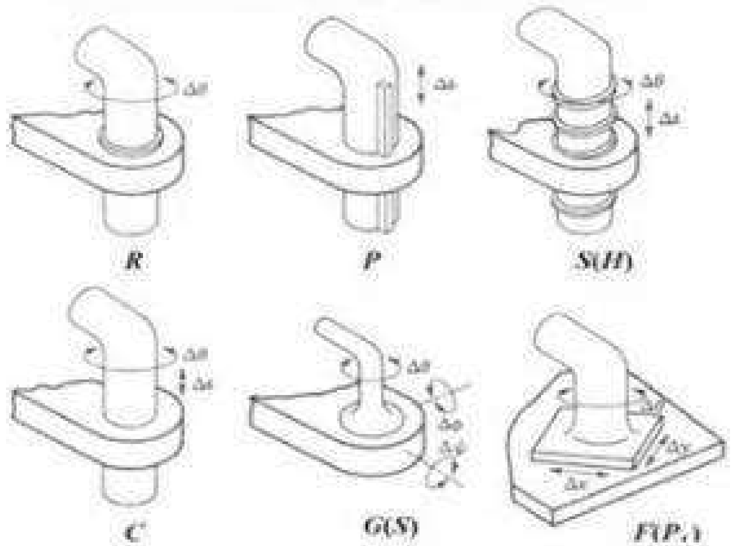




4 bar Mechanism has 1 DoF as the angle turned by the crank AD is fully describing the position of the every link of the mechanism

DOF

The Lower Pairs Joints



Pair	Symbol	Pair Variable	Degree of Freedom	Relative Motion
Revolute	R	$\Delta\theta$	1	Circular
Prism	P	Δs	1	Rectilinear
Screw	$S(H)$	$\Delta\theta$ or Δs ($\Delta s = h\Delta\theta$)	1	Helical
Cylinder	C	$\Delta\theta$ and Δs	2	Cylindric
Sphere	$G(S)$	$\Delta\theta, \Delta\phi, \Delta\psi$	3	Spheric
Flat	$F(P_1)$	$\Delta x, \Delta y, \Delta\theta$	3	Planar

DEGREES OF FREEDOM/MOBILITY OF A MECHANISM

It is the number of inputs (number of independent coordinates) required to describe the configuration or position of all the links of the mechanism, with respect to the fixed link at any given instant.

KUTZBACH CRITERION

For mechanism having plane motion

$$\text{DoF} = n = 3\,(l - 1) - 2j - h$$

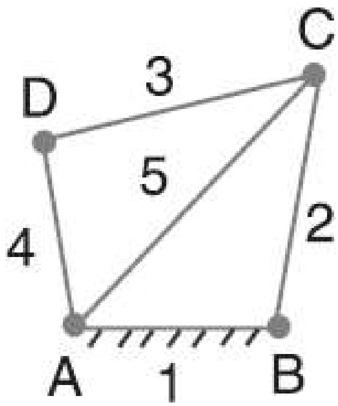
l = number of links

j = number of binary joints or lower pairs (1 DoF pairs)

h = number of higher pairs (i.e. *2 DoF pairs)*

NUMERICAL EXAMPLE -1 &2

Determine the DoF of the mechanism shown below:

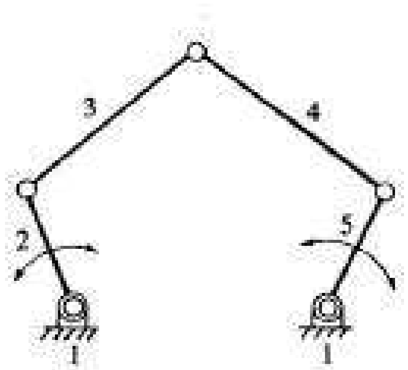


$$n = 3\,(l - 1) - 2\,j - h$$ **Kutzbach Criterion**

l = 5 ; j = 2+ 2* (3-1) = 6 ; h = 0

$$n = 3\,(5 - 1) - 2 \times 6 = 0$$

DoF = 0, means that the mechanism forms a structure



l = 5 ; j = 5; h = 0

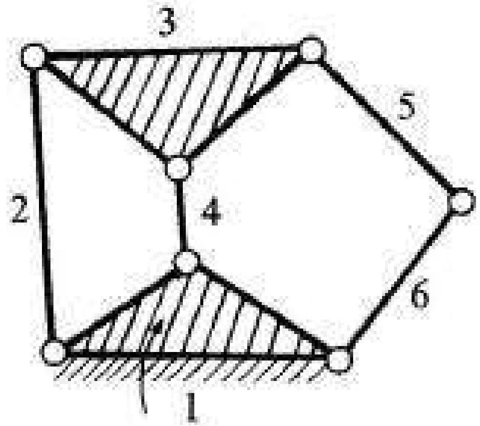
n = 3 (5-1) - 2*5 - 0 = 2

Two inputs to any two links are required to yield definite motions in all the links.

NUMERICAL EXAMPLE -3 &4

Determine the Dof for the links shown below:

$n = 3\,(l-1) - 2\,j - h$ **Kutzbach Criterion**

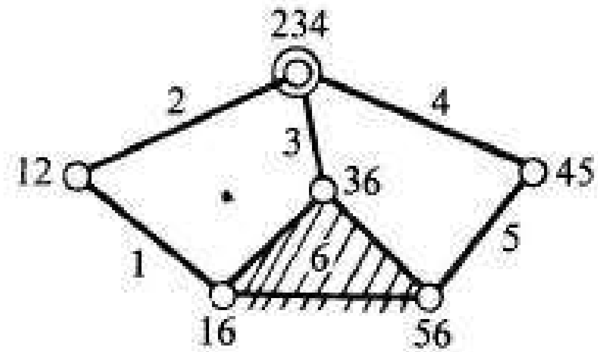


l = 6 ; j = 7 ; h = 0

n = 3 (6-1) - 2 (7) - 0 = 1

Dof = 1

i.e., one input to any one link will result in definite motion of all the links.



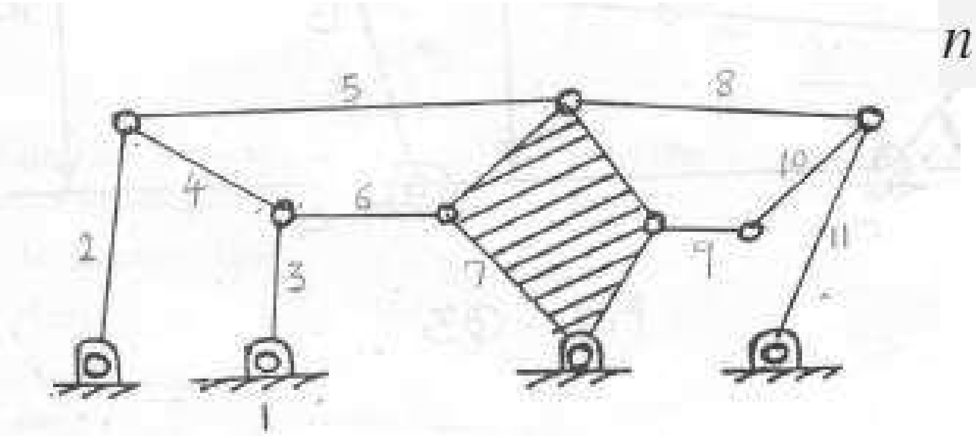
Note: at the intersection of 2, 3 and 4, two lower pairs are to be considered

l = 6 ; j = 5 + 1 (3-1) = 7 ; h = 0

n = 3 (6-1) - 2 (7) - 0 = 1

Dof = 1

NUMERICAL EXAMPLE - 5



$n = 3\,(l - 1) - 2\,j - h$ **Kutzbach Criterion**

l = 11 ; j = 7 + 4 (3-1) = 15 ; h = 0

n = 3 (11-1) - 2 (15) - 0 = 0

Dof = 0

Here, j= 15 (two lower pairs at the intersection of 3, 4, 6; 2, 4, 5; 5, 7, 8; 8, 10, 11) and h = 0.

Summary

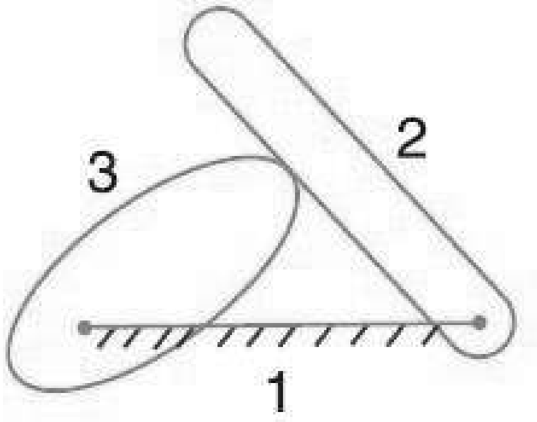
Dof = 0, Structure

Dof = 1, mechanism can be driven by a single input motion

Dof = 2, two separate input motions are necessary to produce constrained motion for the mechanism

Dof = -1 or less, redundant constraints in the chain and it forms a statically indeterminate structure

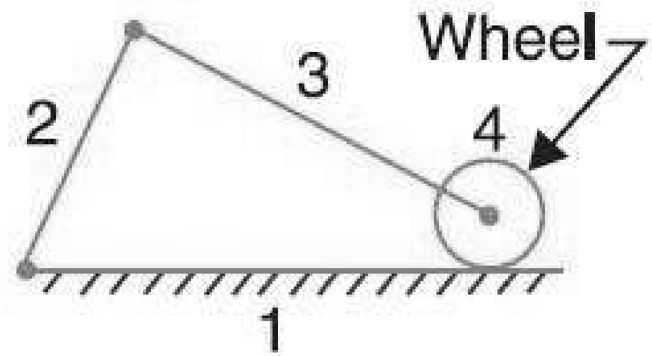
KUTZBACH CRITERION FOR HIGHER PAIRS



$l = 3, j = 2$ and $h = 1$

$$n = 3\,(3-1) - 2 \times 2 - 1 = 1$$

$$n = 3\,(l-1) - 2\,j - h$$

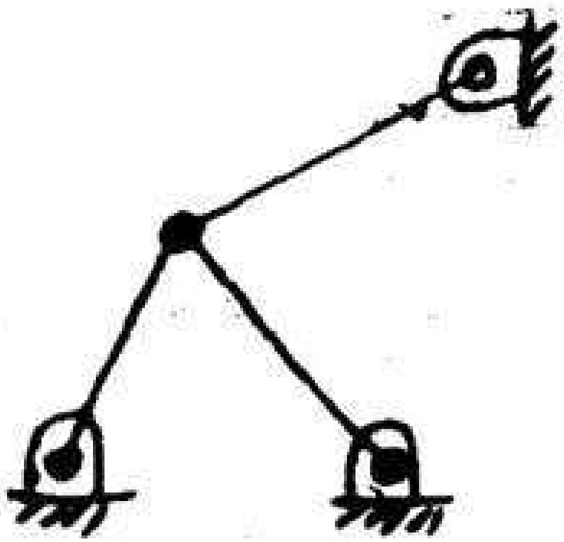


$l = 4,\ j = 3$ and $h = 1$

$$n = 3\,(4-1) - 2 \times 3 - 1 = 2$$

KUTZBACH CRITERION

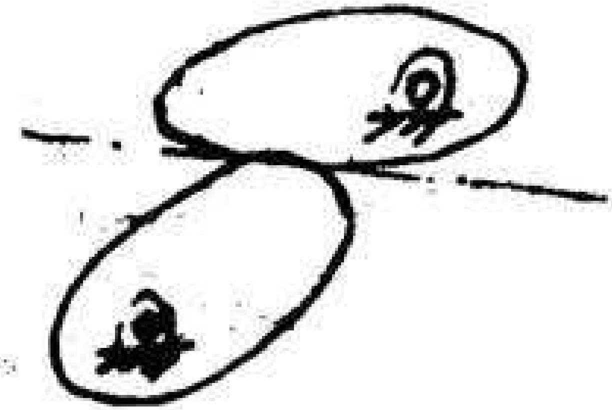
$$n = 3\,(l - 1) - 2\,j - h$$



l = 4, j = 5, h=0

n = 3 (4-1) - 2 (5) - 0 = -1

Indeterminate structure



l = 3, j = 2, h=1

n = 3 (3-1) - 2 (2) - 1 = 1

GRUBLER'S CRITERION FOR PLANE MECHANISMS

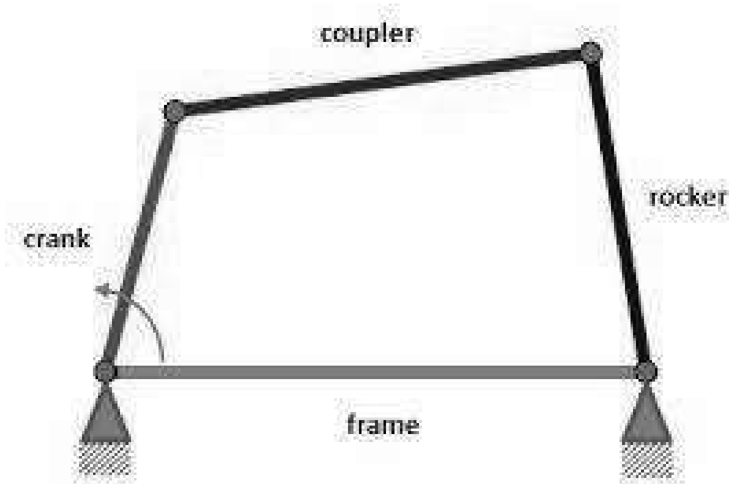
Kutzbach Criterion $n = 3\,(l - 1) - 2\,j - h$

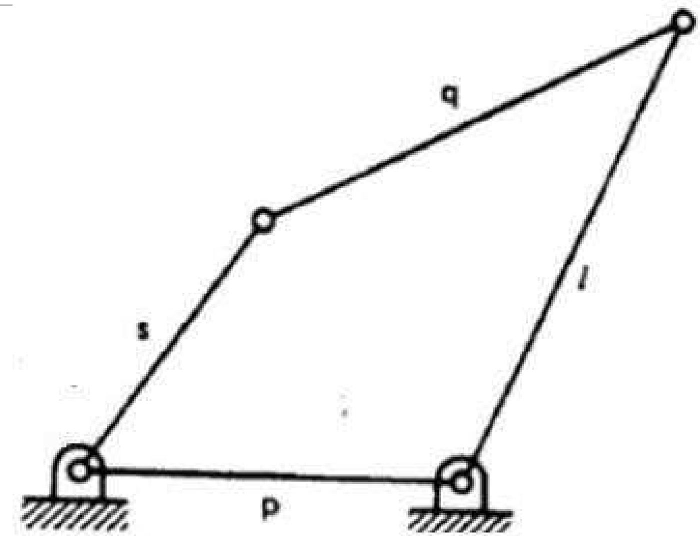
Grubler's criterion applies to mechanisms having 1 DoF.

Substituting n = 1 and h=0 in Kutzbatch equation, we can have Grubler's equation.

$$1 = 3\,(l - 1) - 2\,j \qquad \text{or} \qquad 3l - 2j - 4 = 0$$

GRASHOF’S LAW

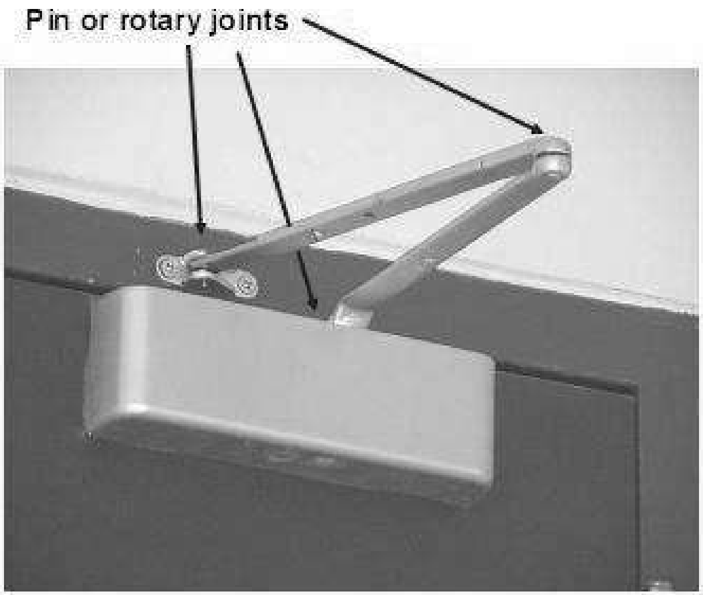


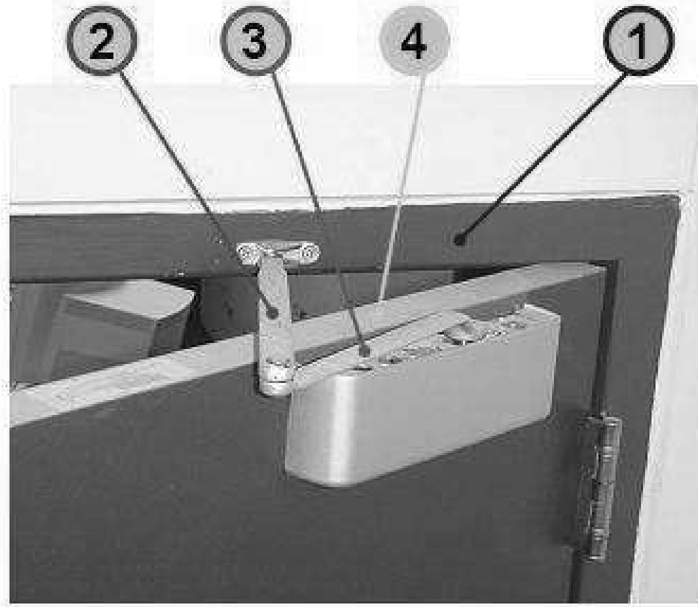


$$l+s \leq p+q$$

According to **Grashof ’s law for a four bar mechanism,** the sum of the shortest and longest link lengths <u>should not be greater than</u> the sum of the remaining two link lengths if there is to be continuous relative motion between the two links.

Example: 4 bar door damper linkage





1	= Wall	or	Link 1	This is the grounded (held still)
2	= Bar 2	or	Link 2	
3	= Bar 3	or	Link 3	
4	= Door	or	Link 4	